Aufgaben Potenzen

1) a) $10^3 \times 10^{-4} = 10^{3-4} = 10^{-1} = \frac{1}{10^1} = \frac{1}{10} = 0{,}1$

b) $10^5 : 10^{-5} = \frac{10^5}{10^{-5}} = 10^5 \cdot 10^5 = 10^{10}$

c) $\frac{0{,}3 \cdot 10^5 + 0{,}7 \cdot 10^5}{10^{-4}} = \frac{1 \cdot 10^5}{10^{-4}} = 10^5 \cdot 10^4 = 10^9$

2) a) $\sqrt{8x^2 + x^2} = \sqrt{9x^2} = \sqrt{9} \cdot \sqrt{x^2} = 3x$

b) $\sqrt[3]{x(20x^2 + 7x^2)} = \sqrt[3]{27x^3} = \sqrt[3]{27} \cdot \sqrt[3]{x^3}$

$= 3x$

c) $\frac{\sqrt{3x^2}}{9} = \frac{\sqrt{3} \cdot \sqrt{x^2}}{9} = \frac{\sqrt{3}}{9} \cdot x$

d) $\left(x^{-\frac{1}{3}}\right)^6 = x^{-\frac{1}{3} \cdot 6} = x^{-2} = \frac{1}{x^2}$

3) a) $3 \cdot (\log(x) - \log(y))$

$= 3 \cdot \log\left(\frac{x}{y}\right) = \log\left(\left(\frac{x}{y}\right)^3\right)$

b) $\frac{\frac{1}{2} \log(x^2)}{\log(x)} = \frac{\log\left((x^2)^{1/2}\right)}{\log(x)} = \frac{\log\left(x^{2 \cdot \frac{1}{2}}\right)}{\log(x)}$

$= \frac{\log(x)}{\log(x)} = 1$

c) $\frac{\log(x)}{\log(e)} - \ln(x) + 1 = \ln(x) - \ln(x) + 1 = 1$

4 b) $3^x = 4^7 \quad | \ln$

$\Leftrightarrow \ln(3^x) = \ln(4^7)$

$\Leftrightarrow x \cdot \ln(3) = 7 \cdot \ln(4) \quad | : \ln(3)$

$\Leftrightarrow x = \frac{7 \cdot \ln(4)}{\ln(3)} = \underline{8{,}83}$

4a)

$4^x = 2^x + 12$ geraten: $x = 2$

$\Leftrightarrow (2^2)^x = 2^x + 12$

$\Leftrightarrow 2^{2x} = 2^x + 12$

$\Leftrightarrow (2^x)^2 = 2^x + 12$ $\quad z = 2^x$

$\Leftrightarrow z^2 = z + 12$

$\Leftrightarrow z^2 - z - 12 = 0$ $\quad z_1 = 4,\ z_2 = -3 \quad |$ Lösung für z

$\Leftrightarrow (z-4)(z+3) = 0$

$z = 2^x \quad | \ln$

$\Leftrightarrow \ln(z) = \ln(2^x)$

$\Leftrightarrow \ln(z) = x \cdot \ln(2) \Rightarrow x = \frac{\ln(z)}{\ln(2)}$

a) $z_2 = -3$

$x = \frac{\ln(-3)}{\ln(2)}$ ↯

b) $z_1 = 4$

$x = \frac{\ln(4)}{\ln(2)} = 2$

4) $K_n = K_0 \cdot (1+i)^n$

gegeben gesucht: n

$K_0 = 1\,€ \quad i = 0{,}03$

$K_n = 3\,€$

$K_n = K_0 (1+i)^n \quad | : K_0$

$\Leftrightarrow \frac{K_n}{K_0} = (1+i)^n \quad | \ln$

$\Leftrightarrow \ln\left(\frac{K_n}{K_0}\right) = \ln\left((1+i)^n\right)$

$\Leftrightarrow \ln\left(\frac{K_n}{K_0}\right) = n \cdot \ln(1+i) \quad | : \ln(1+i)$

6) Fläche Küche

Boden	$5m \times 4m$	$= 20\,m^2$
Decke	$5m \times 4m$	$= 20\,m^2$
Wände	$2 \times 5m \times 3m$	$= 30\,m^2$
Wände	$2 \times 4m \times 3m$	$= 24\,m^2$
		$94\,m^2$

Fläche Amöbe

$$D = 10{,}433 \cdot 10^{-6}\,m$$

$$F = \pi \cdot r^2 = 3{,}141 \cdot \left(\frac{10{,}433 \cdot 10^{-6}\,m}{2}\right)^2 = 8{,}547 \cdot 10^{-11}\,m^2$$

Anzahl Amöben: $N = \frac{94\,m^2}{8{,}547 \cdot 10^{-11}\,m^2} = 1{,}1 \cdot 10^{12}$

Tage	N
0	1
1	2
3	4
4	8

$$N(J) = 2^t \stackrel{!}{=} 1{,}1 \cdot 10^{12} \quad | \ln$$

$$\Leftrightarrow \ln(2^t) = \ln(1{,}1 \cdot 10^{12})$$

$$\Leftrightarrow t \cdot \ln(2) = \ln(1{,}1 \cdot 10^{12})$$

$$\Leftrightarrow t = \frac{\ln(1{,}1 \cdot 10^{12})}{\ln(2)}$$

$$= \underline{40 \text{ Tage}}$$

Gleichungssysteme

Hund H

Leine L

a) $H + L = 110$

b) $H - L = 100$

a)-b) $2L = 10$

$\Rightarrow L = 5$

a) $H + L = 110$

b) $H - L = 100$

a)+b) $2H = 210$

$\Rightarrow H = 105$

Was darf man mit Gleichungen machen, ohne sie zu ändern??

1) 2 Gleichungen addieren/subtrahieren

2) Gleichung mit Konstanten multiplizieren

-> Elementare Umformungen

Beispiel:

$$\begin{aligned} 2x_1 + 2x_2 + 4x_3 &= 18 \\ 2x_1 + x_2 &= 4 \\ x_1 + 2x_2 &= 5 \end{aligned}$$

Schreibfaul

$$\begin{array}{ccc} x_1 & x_2 & x_3 & \\ \end{array}$$
$$\left(\begin{array}{ccc|c} 2 & 2 & 4 & 18 \\ 2 & 1 & 0 & 4 \\ 1 & 2 & 0 & 5 \end{array}\right)$$

$$\begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \left(\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 1 & 4 & 14 \\ 0 & 0 & 1 & 3 \end{array}\right) \end{array} \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}$$

3) $\underline{x_3 = 3}$

2) $x_2 + 4x_3 = 14$

$\underline{x_2 = 2}$

1) $x_1 + x_2 + 2x_3 = 9$

$\Rightarrow x_1 + 2 + 6 = 9$

$\Rightarrow \underline{x_1 = 1}$

$$\left(\begin{array}{ccc|c} \boxed{2} & 2 & 4 & 18 \\ 2 & 1 & 0 & 4 \\ 1 & 2 & 0 & 5 \end{array}\right) :2$$

Setze Pivot-Element nach „links oben"

-> Teile Zeile 1 durch Pivotelement

$$\left(\begin{array}{ccc|c} \boxed{1} & 1 & 2 & 9 \\ 2 & 1 & 0 & 4 \\ 1 & 2 & 0 & 5 \end{array}\right)\begin{array}{l} \\ -2\cdot Z_1 \\ -Z_1 \end{array}$$

-> Erzeuge „0" unterhalb Pivotelement durch Subtraktion von Vielfachen der Pivotzeile

$$\left(\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & \boxed{-1} & -4 & -14 \\ 0 & 1 & -2 & -4 \end{array}\right)\begin{array}{l} \\ :(-1) \\ \\ \end{array}$$

-> Pivotelement wandert auf nächste Position

-> von vorne

$$\left(\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & \boxed{1} & 4 & 14 \\ 0 & 1 & -2 & -4 \end{array}\right)\begin{array}{l} \\ \\ -Z_2 \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 1 & 4 & 14 \\ 0 & 0 & \boxed{-6} & -18 \end{array}\right)\begin{array}{l} \\ \\ :(-6) \end{array}$$

Rekursive LSG

$$\begin{array}{cc} & \begin{array}{cccc} x_1 & x_2 & x_3 & \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \left(\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 1 & 4 & 14 \\ 0 & 0 & 1 & 3 \end{array}\right) \end{array}$$

3) $x_3 = 3$

2) $x_2 + 4x_3 = 14$

$\Rightarrow x_2 + 4\cdot 3 = 14 \Leftrightarrow x_2 = 2$

1) $x_1 + x_2 + 2x_3 = 9$

$\Rightarrow x_1 + 2 + 6 = 9 \Leftrightarrow \underline{x_1 = 1}$

$$\left(\begin{array}{ccc|c} \boxed{1} & 1 & 2 & 4 \\ 2 & 1 & 0 & 8 \\ 1 & 2 & 0 & 1 \end{array}\right) \begin{array}{l} \\ -2\cdot z_1 \\ -z_1 \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 2 & 4 \\ 0 & \boxed{-1} & -4 & 0 \\ 0 & 1 & -2 & -3 \end{array}\right) \begin{array}{l} \\ :(-1) \\ \\ \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 2 & 4 \\ 0 & \boxed{1} & 4 & 0 \\ 0 & 1 & -2 & -3 \end{array}\right) \begin{array}{l} \\ \\ -z_2 \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 2 & 4 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & \boxed{-6} & -3 \end{array}\right) \begin{array}{l} \\ \\ :(-6) \end{array}$$

$$\begin{array}{c} \\ (1) \\ (2) \\ (3) \end{array} \begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \left(\begin{array}{ccc|c} 1 & 1 & 2 & 4 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 1 & 1/2 \end{array}\right) \end{array}$$

(3) $\underline{x_3 = 1/2}$

(2) $x_2 + 4x_3 = 0$

$\Rightarrow x_2 + 2 = 0 \Rightarrow \underline{x_2 = -2}$

(1) $x_1 + x_2 + 2x_3 = 4$

$\Rightarrow x_1 - 2 + 1 = 4 \Leftrightarrow \underline{x_1 = 5}$

$$\left(\begin{array}{ccc|c} \boxed{2} & 2 & 4 & 18 \\ 2 & 1 & 0 & 4 \\ 2 & 3 & 4 & 22 \end{array}\right) :2$$

$$\begin{array}{c} \left(\begin{array}{ccc|c} \boxed{1} & 1 & 2 & 9 \\ 2 & 1 & 0 & 4 \\ 2 & 3 & 4 & 22 \end{array}\right) \end{array} \begin{array}{l} \\ -2 \cdot Z_1 \\ -2 \cdot Z_1 \end{array}$$

$$\begin{array}{c} \left(\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & \boxed{-1} & -4 & -14 \\ 0 & 1 & 0 & 4 \end{array}\right) \end{array} \begin{array}{l} \\ :(-1) \\ \\ \end{array}$$

$$\begin{array}{c} \left(\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & \boxed{1} & 4 & 14 \\ 0 & 1 & 0 & 4 \end{array}\right) \end{array} \begin{array}{l} \\ \\ -Z_2 \end{array}$$

$$\begin{array}{c} \left(\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 1 & 4 & 14 \\ 0 & 0 & \boxed{-4} & -10 \end{array}\right) \end{array} \begin{array}{l} \\ \\ :(-4) \end{array}$$

$$\begin{array}{c} \\ (1) \\ (2) \\ (3) \end{array} \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & x_3 & \end{array} \\ \left(\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 1 & 4 & 14 \\ 0 & 0 & 1 & 5/2 \end{array}\right) \end{array}$$

(3) $x_3 = 5/2$

(2) $x_2 + 4x_3 = 14$

$\Rightarrow x_2 + 10 = 14 \Rightarrow x_2 = 4$

(1) $x_1 + x_2 + 2x_3 = 9$

$\Rightarrow x_1 + 4 + 5 = 9 \Rightarrow \underline{x_1 = 0}$